Toronto District School Board

# TEACHING AND LEARNING MATHEMATICS RESEARCH SERIES I:
# *Effective Instructional Strategies*

**Research & Information Services**
Toronto District School Board
November 2016
Report No. 16/17-08

**TITLE:** Teaching and Learning Mathematics Research Series l: Effective Instructional Strategies
**AUTHORS:** Erhan Sinay and Ashley Nahornick



**Cite as:**
Sinay, E., & Nahornick, A. (2016). *Teaching and learning mathematics research series l: Effective instructional strategies.* (Research Report No. 16/17-08). Toronto, Ontario, Canada: Toronto District School Board.



Research & Information Services
Toronto District School Board
1 Civic Centre Court, Lower Level
Etobicoke, ON M9C 2B3

Fax: 416-394-4946



R02(Renewed Mathematics Strategy\Research\Series I\Teaching and Learning Math Research Series 1)es.1485

# TABLE OF CONTENTS

## TABLES



## FIGURES

## ACKNOWLEDGEMENTS

We would like to thank and acknowledge the support and contributions of the following reviewers in this study:

**Thomas G. Ryan**, Professor, Faculty of Education, Nipissing University

**Kamini Jaipal-Jamani,** Associate Professor, Department of Teacher Education Brock University, Faculty of Education

**Margaret Douglin,** Research Coordinator, Research & Information Services, Toronto District School Board

## BACKGROUND AND RATIONALE

Mathematics is foundational in many ways that informs our decisions in areas of our lives. Teaching and learning mathematics is at the heart of education. Learning mathematics aims to link school to everyday life, provide skill acquisition, prepare students for the workforce, and foster mathematical thinking (Ontario Ministry of Education, 2005). Mathematics involves learning to problem-solve, investigate, represent, and communicate mathematical concepts and ideas, and making connections to everyday life (Ontario Ministry of Education, 2005). However, there is a concern that Canadian students' mathematical achievements are on the decline (EQAO, 2012). In 2012, the Programme for International Student Assessment (PISA), an international study examining 15-year-old students' mathematical performance among 60 countries or economies, showed that math scores in Ontario have declined by 16 percentage points over the last nine years (EQAO, 2012).

Historical results of Education Quality and Accountability Office (EQAO) assessments also showed considerable decline in mathematics achievement overtime. Over the past five years (2011-12 to 2015-16), Ontario's Grade 3 students' results declined by five percentage points, from 68% to 63% and Grade 6 students' results declined by eight percentage points from 58% to 50%. The province had the lowest improvement in Grade 6 mathematics compared to any of the other EQAO assessments since 1998-99 (4 percentage points from 46% to 50%). For all of the other assessments, there were 10 to 34 percentage point improvements since the test was administered in 1997-98[1].

Additionally, the Trends in International Mathematics and Science Study (TIMSS) conducted in 2011, which examined mathematics and science achievements for students in Grades 4 and 8, demonstrated that only 33% of students in Grade 4 and 32% of students in Grade 8 indicated high levels of confidence in math (EQAO, 2011). Mathematical skills and confidence are essential for students. Given the importance of mathematical skills and confidence, this study focuses on research-based instructional strategies that can provide guidance for effective classroom practices for supporting student development in mathematics.

### Create a Supportive and Engaging Classroom Environment

A supportive and engaging classroom environment is important to help develop students' mathematical understanding and confidence (Ontario Ministry of Education, 2005). The classroom needs to be a place of investigation, where students do meaningful mathematics in a safe and positive space. "Meaningful mathematics takes place in K to 12 classrooms that support students as they investigate, represent and connect mathematical ideas through discussion in the context of problem solving" (Suurtamm et al., 2015).

---

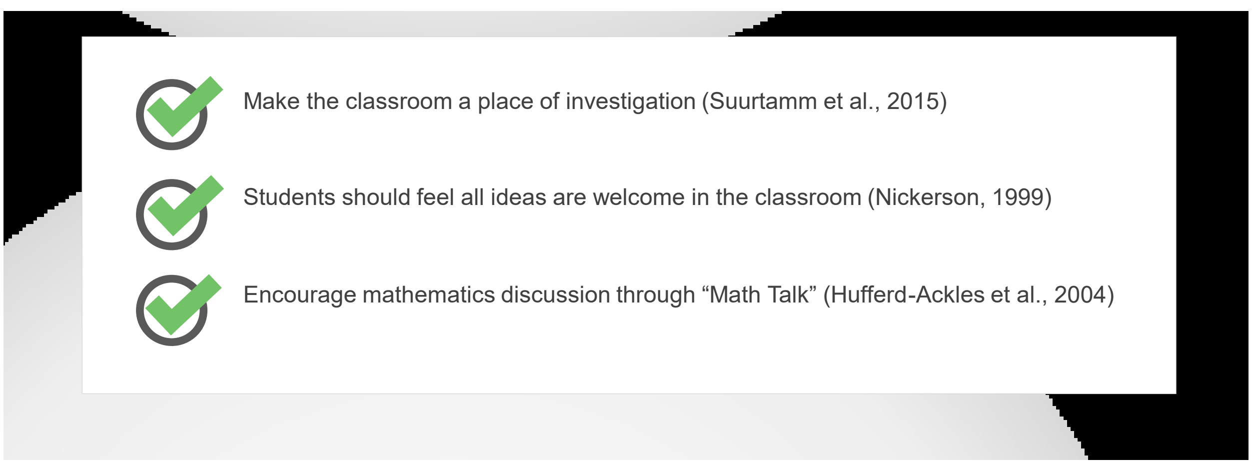
[1] These results were gathered from historical reports provided by EQAO. EQAO Grade 3 assessments started in 1996-97, Grade 6 in 1998-99, Grade 9 Assessment of Mathematics in 2000-01; and the first Ontario Secondary School Literacy Test (OSSLT) was administered as a pilot in 2000-01.

The classroom needs to be made a place of investigation by supporting unusual ideas and responses by students (Feldhusen & Treffinger, 1985; Nickerson, 1999; Sternberg & Williams, 1996). Bruce (2007), who studied ways to improve students' educational experience in mathematics, explains that discussion in the mathematics classroom is very important. The mathematics classroom should feel like a community where ideas can be discussed, developed, debated, and understood (Bruce, 2007). Students should feel that all ideas are welcome in the classroom, even those that are unconventional (Nickerson, 1999).

One way to encourage classroom dialogue in mathematics is through "Math Talk." Math Talk is a way to have structured mathematical discussions that construct knowledge and meaning (Hufferd-Ackles, Fuson, & Gamoran Sherin, 2004). Students become the co-constructors of knowledge through asking questions, justifying their work, and communicating their ideas to each other (Wagganer, 2015). Students are asked to explain their ideas, justify their thinking, and question one another on their work and compare and contrast ideas and solutions (Suurtamm et al., 2015).

Teachers guide and extend students' learning through mathematics discussion questions, sentence stems (sometimes called sentence starters), asking for examples, and asking for justifications of work (Wagganer, 2015). Bruce (2007) suggests giving students sufficient time to do math talk, and not to intervene with probing questions or ideas too quickly. Additionally, Bruce (2007) suggests math talk could be part of the classroom assessment.

**Figure 1: Recommendations for Creating a Supportive and Engaging Classroom**




## Provide Strong Mathematics Foundations in the Early Years

More and more jurisdictions, teacher associations, and researchers (Duncan et al., 2007; Ontario Ministry of Education, 2003; National Association for the Education of Young Children [NAEYC] & National Association of Early Childhood Specialists in State Departments of Education (NAECS/SDE 2003; National Research Council, 2009; Student Achievement Division, 2011) continue to advocate for providing our very young children with a strong mathematics foundation. Providing our youngest learners with a strong mathematics foundation can play an important role in future academic achievement. Researchers have found that early years' math skills can have a strong predictive power for future academic achievement (Duncan et al., 2007). Furthermore, the Canadian Child Care Federation (2010) has found that most differences in children's numeracy skills in school can be linked to the opportunities children had at home and early childhood environments. As such, the National Council of Teachers of Mathematics [NCTM] (2013) strongly advocates for greater access to pre-kindergarten programs for all students.

### Importance of Early Years' Mathematics

Young children, even infants, need to be stimulated and challenged mathematically. "Claims that children are either too young, in the wrong stage or not ready to learning something have been shown time and time and again to be wrong" (National Mathematics Advisory Panel [NMPA], 2008, p.13). Young children need to be stimulated at home and/or through early childhood education environments by thinking and talking about numbers, discussing mathematics around them, looking for patterns, and classifying similarities and differences of objects (Berger, n.d). There is no age too early to learn math. It has been shown that children as young as infants can tell differences between small quantities, and that even before kindergarten, children are able to look for patterns and discuss similarities and differences in quantity (Canadian Child Care Federation, 2010).

### Create Appropriate Curriculum Standards

Research supports creating standards based on early childhood mathematics education (NAEYC & NAECS/SDE, 2003; National Research Council, 2009; National Council of Teachers of Mathematics [NCTM], 2013). This includes creating curriculum standards, resources, and challenging mathematics activities directed to young learners. The NCTM (2013) advocates that "young children in every setting should experience mathematics through effective, research-based curricula and teaching practices" (p. 1).

The NCTM (2013) recommends that early years' mathematics education should focus on four big ideas: (1) numbers and operations, (2) geometry, (3) algebraic reasoning, and (4) measurement. The Ontario Ministry of Education Early Math Strategy recommends having students focus on patterns, shapes, comparisons, number sense, and problem-solving (Ontario Ministry of Education, 2003). Nonetheless, the overarching goal is to ensure our youngest learners are receiving challenging and stimulating mathematics learning environments. Balfanz (1999) contends that often the curriculum for early years is insufficient since young students bring more mathematical knowledge to school than we think.

### Develop Knowledge of Children's Developmental Process

It is important for educators to be mindful of the developmental process and unique characteristics of young learners (Student Achievement Division, 2011). Young learners go through different stages of development as they grow and as a result, early years practitioners need to have a strong understanding of the development and characteristics of young learners (Student Achievement Division, 2011). For example, from ages 3-6, children learn to formalize their thinking by using their knowledge, experiences, and vocabulary to mathematize their ideas (Berger, n.d). Since young children do not see things in the same way as adults, early childhood educators need to understand the child's point of view, and not impose their thinking (Student Achievement Division, 2011).

### Implement Appropriate Classroom Activities

Young learners are curious and playful when it comes to mathematics, and learn by doing, thinking, and talking (Ontario Ministry of Education, 2003). They do not memorize mathematics. Early childhood learning should focus on capitalizing on this inquisitive behavior by encouraging curiosity and investigations (Berger, n.d). Free play should be encouraged in early years' mathematics education by focusing to maximize students' mathematical learning, for example asking students probing questions during play time to promote student thinking. Additionally, students should do more than play (Berger, n.d). They should be pushed to discuss mathematics, use appropriate mathematics vocabulary and extend their learning (Berger, n.d). Early years' educators should encourage "Math Talk" to foster mathematical dialogue (Student Achievement Division, 2011).

Other instructional recommendations include: (1) using problems that have meaning for young learners, not problems that hold meaning to you as an adult, (2) allowing for creativity, (3) encouraging and supporting different problem-solving methods, (4) scaffolding, and (5) helping with creating connections (Student Achievement Division, 2011).

### Teach Mathematics in an Interdisciplinary Manner

It is recommended that early years educators teach mathematics in an interdisciplinary manner, connecting mathematics to other subjects (NCTM, 2013). The National Council of Teachers of Mathematics (1995) argues that K-4 teaching should be interdisciplinary in nature and be organized by questions, burning issues, problems or projects that connect mathematics, science and reading. Nikitina and Mansilla (2003) explain that mathematics is often taught in isolation from other subject areas, creating a lack of motivation for students to learn mathematics. The NCTM (1995) recommends teaching mathematics concepts as part of interdisciplinary units to foster student interest, create authentic connections, and real-life connections.

### Maintain Positive Attitudes about Mathematics

Another important component of quality math instruction for the early years is to inculcate in learners a positive attitude about mathematics. The Ontario Ministry of Education (2003) maintains that "a positive attitude towards mathematics, an understanding of key concepts, and mathematical skills must be developed in the early grades" (p.1). Students can be very much affected by the attitudes and beliefs of their teachers. Teachers should promote positive attitudes and actions about mathematics (Ontario Ministry of Education, 2003).

## Be Knowledgeable about Mathematics

Early childhood educators need to be knowledgeable about mathematics (Student Achievement Division, 2011). As teachers we must be able to explain what we are doing and why in mathematics. Ball (2008) explains that teacher content knowledge can play an important role in student achievement. Teachers need to know mathematics content, underlying reasons behind the mathematics and curriculum expectations (Student Achievement Division, 2011). NCTM (2013) suggests that early childhood educators attend professional development on mathematics content and teaching of math.

## Early Years Strategies and Programs from Other Jurisdictions

Early years education sets the stage for learning, and as such, many jurisdictions are creating strategies, programs, and interventions to improve numeracy skills among early years' students. Following are descriptions of some intervention programs by jurisdiction, along with links for additional information.

### *Australia*

Meiers, Reid, McKenzie, and Mellor (2013) prepared a detailed evidence-based report on literacy and numeracy interventions in the early years (K-3) for the Ministerial Advisory Group on Literacy and Numeracy. The report includes discussion of interventions used in Australian schools, focusing on instructional practices in math, early intervention protocols, professional learning, effective assessment, and developing teachers' understanding of how children learn math. The report includes numerous ideas for numeracy interventions and easy to understand information on the target population, appropriate grade level, intervention focus, and evidence from studies across Australia. For more information, go to: http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc

Dooley, Dunphy, and Shiel (2014) produced a detailed research report for the National Council for Curriculum and Assessment (ages 3-8) in Australia. Their report discusses mathematics pedagogy for the early years, recommended teacher practices, curriculum recommendations, effective teacher preparation and creating partnership with parents. For more information, go to: http://ncca.ie/en/Publications/Reports/NCCA_Research_Report_18.pdf

### *Prince Edward Island - Canada*

The Prince Edward Island Department of Education (2012) published a report on their province-wide early intervention program in numeracy. The report details a pilot study for numeracy intervention with at-risk students in Grades 1-3 using mathematics consultants. For more information, go to: http://www.gov.pe.ca/photos/original/eecd_ENIP201011.pdf

### *United Kingdom*

The United Kingdom Department of Education (2012) has put forth a document outlining numeracy catch-up strategies for underperforming students. The report goes into detail about each of the recommended strategies. The document discusses numeracy interventions, classroom practice strategies, and pedagogy and approaches to support low achieving students. For more information, go to: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/268031/literacy_and_numeracy_catch_up_strategies_in_secondary_schools.pdf

Giford (2014) authored an article for the Advisory Committee on Mathematics Education effective and appropriate early years mathematics pedagogy, and ways to improve success in mathematics. For this article, go to: https://nrich.maths.org/11441

**Figure 2: Provide a Strong Early Years Mathematics Foundation**

# EFFECTIVE TEACHING STRATEGIES FOR ALL GRADES

## Teach for Conceptual Understanding

There is growing evidence that students develop procedural fluency in mathematics, but have difficulty with conceptual understanding (Lawson, 2007). Procedural fluency is the ability to apply and use steps or strategies for different problems (NCTM, n.d), while conceptual understanding is the knowledge of abstract ideas (Rittle-Johnson & Schneider, 2014). Researchers stress the importance of teaching for conceptual understanding (Lawson, 2007; Protheroe, 2007). However, mathematics classes often focus on drills and procedural understanding (Protheroe, 2007), even though "the curriculum is designed to help students build the solid conceptual foundation in mathematics that will enable them to apply their knowledge and further their learning successfully" (Ontario Ministry of Education, 2005 p. 4). Memorizing mathematics is not sufficient for developing understanding.

Students need to be taught to understand the concepts behind procedures (NCTM, n.d). Hiebert (1999) explains that once students learn a procedure, they often have little motivation to learn the underlying concepts behind the procedures. As such, the NCTM (n.d) recommends that students learn the concepts behind the procedures either before or during the instruction of procedures, not after.

Some additional recommended teaching practices for teaching conceptual understanding include:

- Students should be asked to connect procedures to underlying concepts
    - For example, when students are learning about division, explain why they cannot divide by zero (Wathall, 2016)
- Students should be asked to justify their processes (NCTM, n.d) and give self-explanation (Rittle-Johnson & Schnieder, 2014)
- Students should be pushed to work on interesting problems that allow for student generated solutions (Lawson, 2016), facilitate multiple representations, and promote student interaction (Suurtamm et al., 2015).
- Students should compare alternate solution methods (Rittle-Johnson & Schnieder, 2014)
- Students should compare incorrect procedures (Rittle-Johnson & Schnieder, 2014)
- Students need time to work on unfamiliar problems prior to instruction (Rittle-Johnson & Schnieder, 2014)

Figure 3: Teach for Conceptual Understanding


## Teach Reasoning Skills

Research suggests that improving students' reasoning may improve their mathematical abilities (Rittle-Johnson & Jordan, 2016). Reasoning skills include such things as identifying similarities and differences, size, or shape. Kidd, Carlson, Gadzichowski, Boyer, Gallington, and Pasnak (2013) studied students in Grade 1 who had low patterning skills and who were randomly selected for instruction on learning about patterns or numeracy, and found that students who had instruction on patterns outperformed students who received general numeracy instruction. Overall, directly teaching reasoning skills in mathematics may help develop students' mathematical abilities.

Figure 4: Teach Reasoning Skills

## Promote Problem-solving

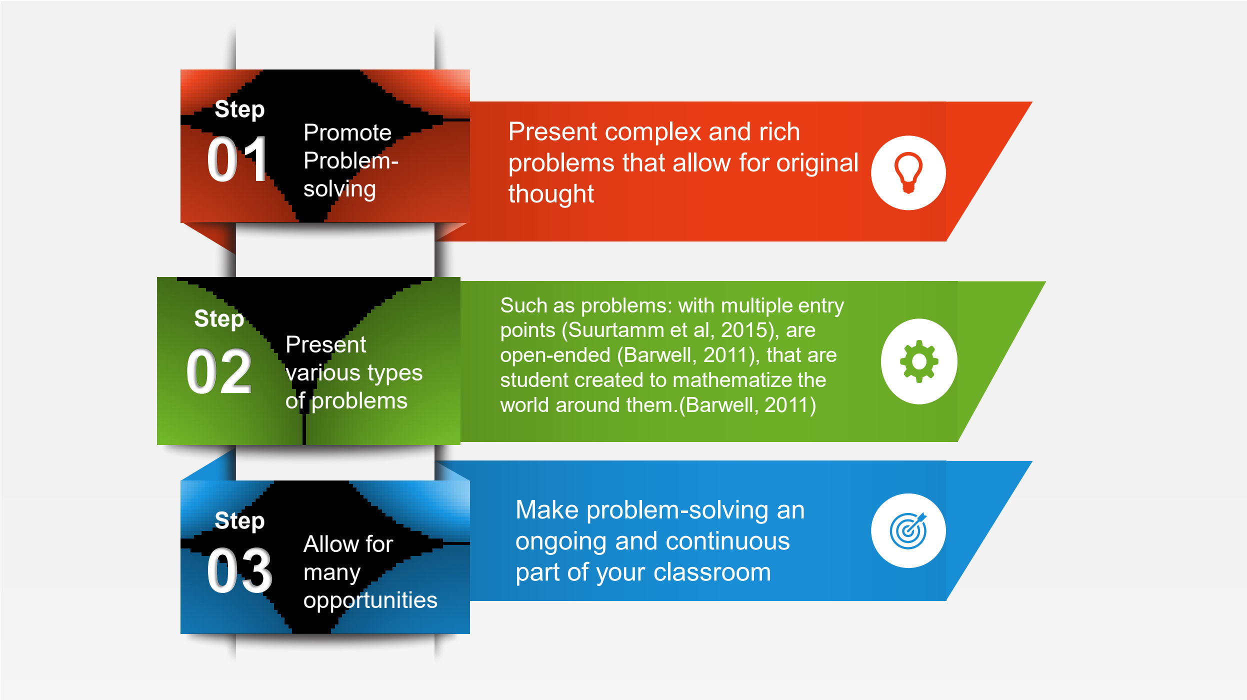
Problem-solving is a foundational building block for learning mathematics. "By learning to solve problems and by learning through problem-solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding" (Ontario Ministry of Education, 2005, p. 11). When the term problem-solving is used, it means that a solver is working on a question where the solver does not know a direct path to achieve the goal (Polya, 1945). Problem-solving is all about coming up with original thoughts, not about practicing drills or algorithms (Polya, 1945). According to Beigie (2008) "a mathematical question may be sophisticated, but it is not a problem unless the student makes real decisions on how to construct the solution" (p. 353). As such, a problem exists when the method of solving is unknown and a solution is necessary (Dossey, McCrone, & O'Sullivan, 2006).

Students need ongoing and continuous opportunities to work on interesting and rich mathematical problems. Many studies suggest that questions labelled as problems in school disregard reality, are rigid, and focus too heavily on procedural fluency. A study of international research on problem-solving found that students often disregard reality when solving problems because the questions are just a way to practice a procedure or algorithm (Greer, 1997). According to Suurtamm et al. (2015) who examined ways to make space for students to think mathematically, it is important to present problems in the mathematics classroom that are complex and rich, allowing for multiple entry points, different approaches, scaffolding, and engagement without imposed procedural steps.

Several researchers (Barwell, 2011; Hoffman & Brahier, 2008; Suurtamm et al., 2015) suggest having students solve open-ended problems. Open-ended problems are questions that allow for a variety of problem-solving methods and a variety of answers. According to Suurtamm et al. (2015), open-ended problems allow students to critically think and improve sense-making because they are forced to work with unfamiliar situations without a guided solution method and procedure to follow.

Other research-based recommendations related to problem-solving include allowing students to create and solve their own problems (Barwell, 2011), work on problems mirroring authentic real-life scenarios (Archbald & Newmann, 1988), compare multiple ways to solve problems (Rittle-Johnson & Jordan, 2016), critique common mistakes (Rittle-Johnson & Jordan, 2016), and mathematize the world around them (Barwell, 2011).

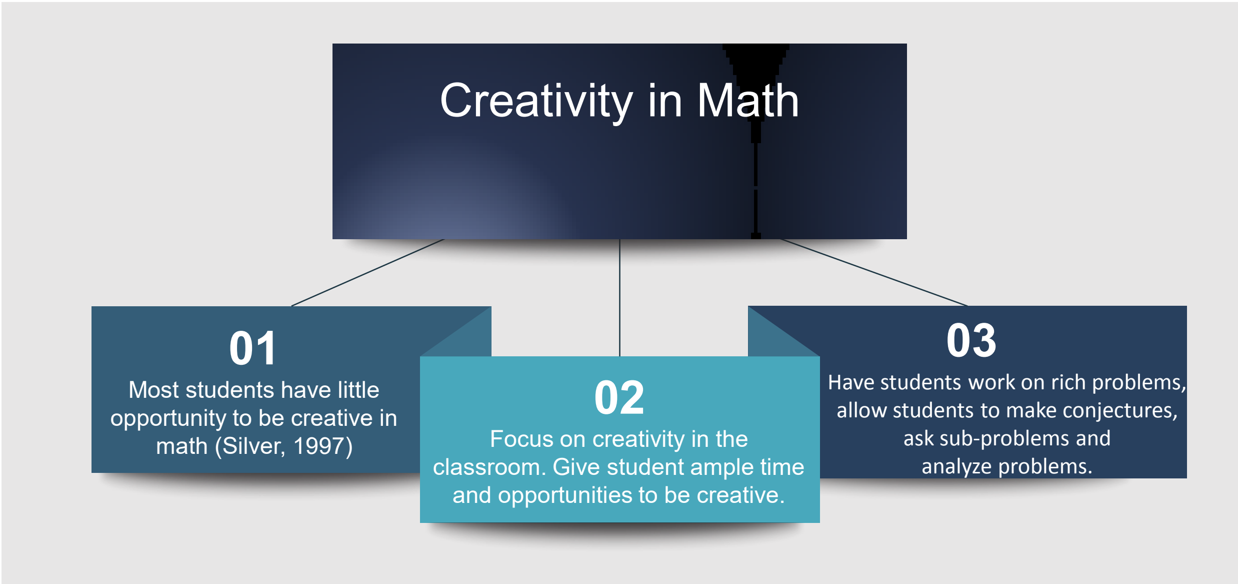
**Figure 5: Promote Problem-solving**




## Creativity in Mathematics

Thinking creatively is at the heart of mathematics (Guilford, 1959); however, most students have little opportunity to experience creativity in mathematics (Silver, 1997). Creative thinking can be developed my making the classroom a place of investigation. Quite often mathematics instruction focuses on drills and algorithms. As such, students often think that mathematics is black and white, with right and wrong answers, and questions that can be solved in 1-2 steps. Whitcombe (1988) explains that classroom environments perpetuate these ideas by focusing only on practical applications of mathematics, presenting boring material and having a rushed learning schedule. Mathematics teaching should focus on mathematical thinking and creativity. Hoffman and Brahier (2008) discuss these ideas in light of past TIMSS results and suggest having students work on rich and complex problems, such as open-ended problems that allow students to make conjectures, ask sub-problems, critically analyze situations, and work with unfamiliar situations. Creativity takes time and experience to develop (Pehkonen, 1997). Students should be given ample time to grapple with stimulating problems (Burns, 1992). Mathematics is not what is seen in textbooks; mathematics is much more than drill-like activities, and students need to be given time and multiple opportunities to work on problems that require them to be resourceful and creative (Pehkonen, 1997).

**Figure 6: Creativity in Mathematics**




## Encourage and Support Collaboration in Mathematics

Mathematics collaboration needs to be encouraged and supported in the classroom. Collaboration is an important way to foster mathematical understanding and increased confidence in mathematics (MacMath, Wallace, & Xiaohong, 2009). By providing a classroom environment where students feel comfortable to collaborate, share, explore, and think mathematically, mathematical confidence can improve (Suurtamm et al., 2015). Substantial research exists explaining that collaboration improves self-confidence (Evans & Dion, 1991), feelings of unity (Evans & Dion, 1991), improved satisfaction (Tett & Meyer, 1993), and cohesiveness (Evans & Dion, 1991).

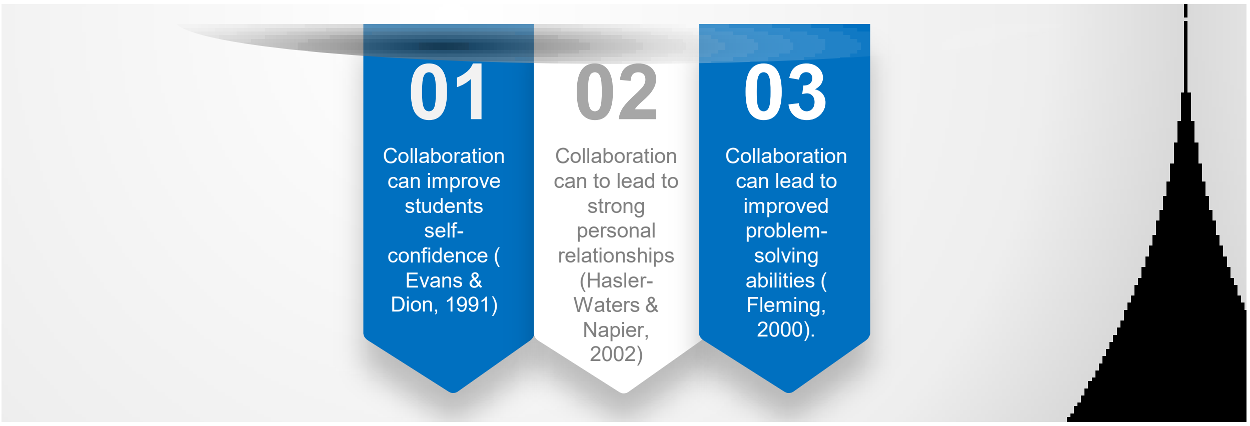
Collaborative work can lead to increased commitment and increased feelings of unity. Lewin (1943), the pioneer of group dynamics, explains that when people have a common goal, it is more likely to be achieved. During World War II, Lewin researched this phenomenon, by testing whether people were more likely to eat undesirable food when they were in a group, and indeed they were (Lewin, 1943). Similarly, promoting collaboration in the mathematics classroom may lead to increased commitment and unity (Suurtamm et al., 2015).

Another benefit of collaboration is its effect on problem-solving. Small group brainstorming during the problem-solving process can have positive effects on later stages of the problem-solving process. Additionally, groups who brainstormed together benefitted during the later steps of the problem-solving process due to increased coordination and cohesion when problem-solving (Fleming, 2000).

Collaboration can also help with creating strong personal relationships. In the classroom environment, Hasler-Waters, and Napier (2002) suggest that collaboration allows students to leverage the strength of their classmates, build relationships, and learn teamwork. An innovative approach recommended by the Institute of Education Services at the U.S.

Department of Education suggests using technology to support students' collaboration (Rittle-Johnson & Jordan, 2016). This could include computer-based group problem-solving, online discussion boards, and online mathematics programs.

**Figure 7: Encourage and Support Collaboration in Mathematics**




## Provide an Inquiry Environment

Many educators and researchers advocate for an inquiry-based learning environment (Suurtamm et al., 2015). An inquiry-based learning environment occurs when students take an active role in learning, and the teacher's role becomes more of a curator of the learning. Inquiry is a way for students to discover mathematics, making mathematics more than just skills and procedures (PRIMAS, 2011). In other words, inquiry-based learning "is a way of teaching and learning mathematics and science in which students are invited to work in the way mathematicians and scientists work" (PRIMAS, 2011, p. 10).[2]

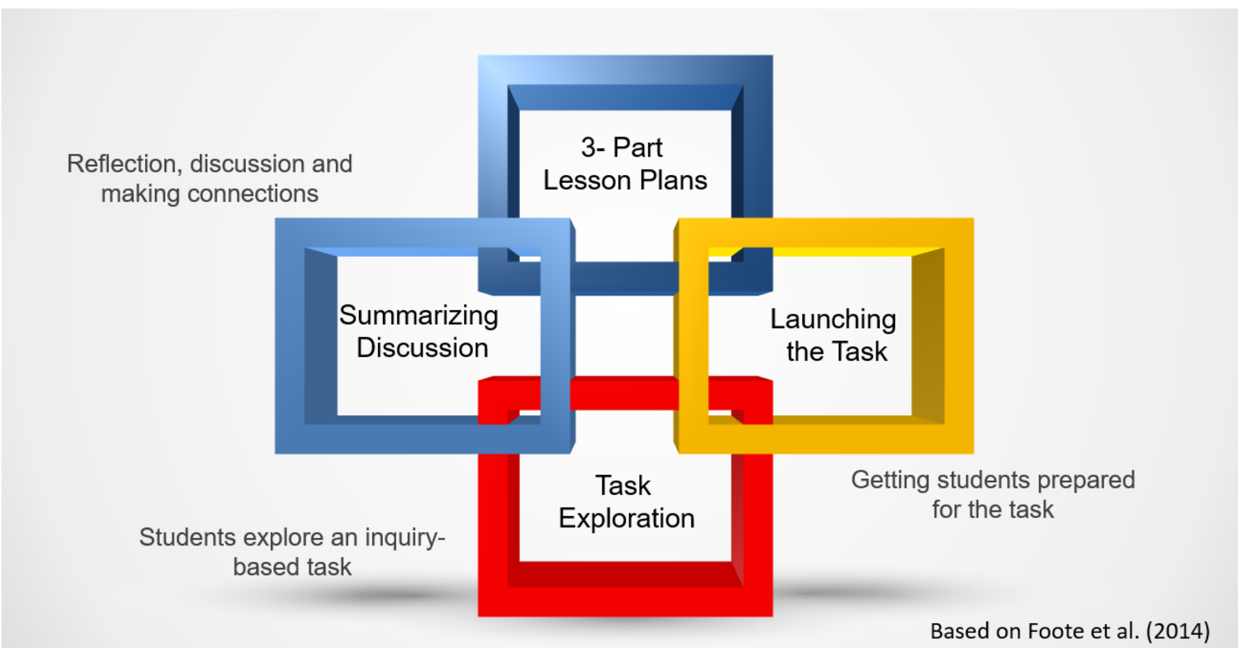
**Figure 8: Provide an Inquiry-based Environment**


[2] To learn more about inquiry-based learning, the Student Achievement Division in Ontario has published a Capacity Building Series on Inquiry-based learning which can be found at the following link: http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_InquiryBased.pdf

## Use Three-Part Mathematics Lesson Plans

Three-part mathematics lessons are a lesson planning method focused on inquiry-based learning supported by the Ontario College of Teachers and National Council of Teachers of Mathematics (Foote, Earnest, Mukhopadhyay, & Curcio, 2014; Ontario College of Teachers, n.d). The three-part math lesson planning technique is supported by the NCTM as an effective way to teach mathematics curriculum standards, support differentiated learning, create an inclusive classroom, promote problem-solving, and engage students in mathematics (Foote et al., 2014). The idea of three-part lessons is to have a hands-on learning experience where the focus is on problem-solving and critical thinking, rather than memorization (Foote et al., 2014). The structure of the three-part lesson includes three components: The first component: "Launching the Task", focuses on getting students excited and cognitively prepared for the lesson. Thereafter, "Task Exploration" is where students explore a mathematics expectation that gives them an opportunity to think, explore, and ask question (Foote et al., 2014). During the "Task Exploration" stages, students often work in small groups while the teacher circulates (Foote et al., 2014). The third stage is "Summarizing Discussion" where the students reflect, discuss, and critically think about their learning. This is the stage where students make connections and link ideas (Foote et al., 2014).

**Figure 9: Three-Part Mathematics Lesson Plans**




## Teaching and Learning with Technology

Teaching and learning with technology is an important part of the mathematics classroom. According to the National Council of Teachers of Mathematics (2011a), "it is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication" (p. 1). Several studies (Gadanidis & Geiger, 2010; Kastberg & Leatham, 2005) have demonstrated that utilizing technology when teaching mathematics can support students in procedural skills, problem-solving, and reasoning. In a meta-analysis of research on the impact of technology on learning, it was found that students who used technology in their classroom perform 12 percentage points higher than those who do not (Tamin, Bernard, Borokhovski, Abrami, & Schmidt, 2011). Different examples of technology in mathematics include: online assessment tools, online collaboration tools, computer algebra systems, apps, calculators, computer applications, and interactive whiteboards (Tamin et al., 2011).

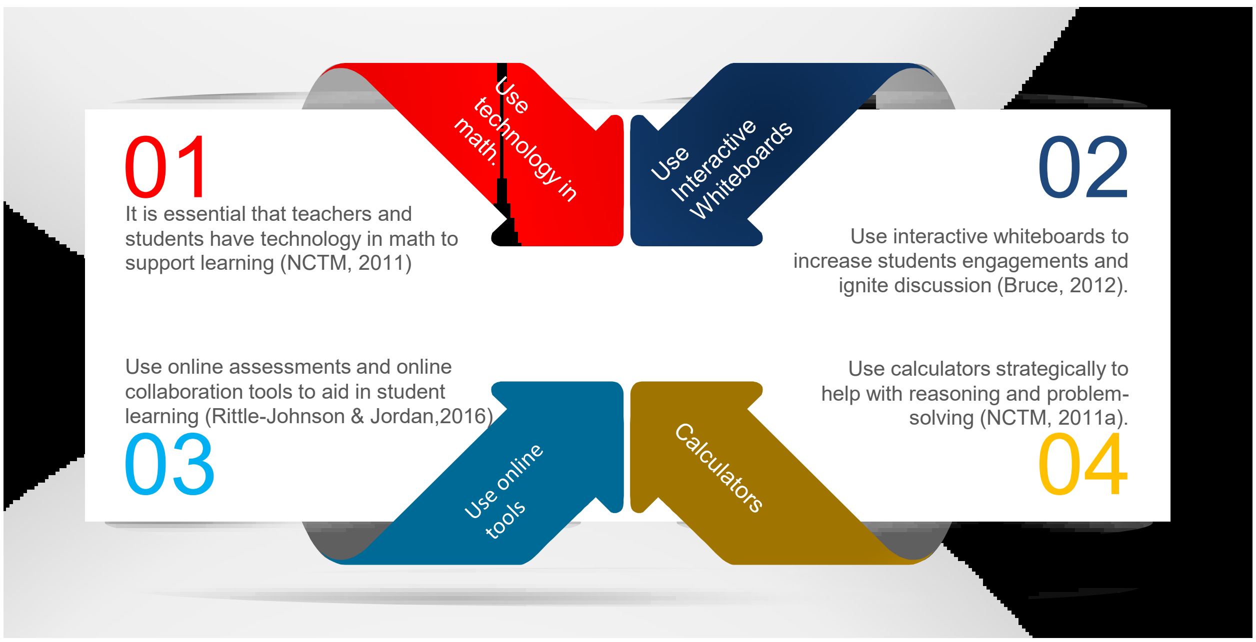
Online assessments are one technology tool that can be useful in teaching and learning mathematics (Rittle-Johnson & Jordan, 2016). Online assessments are a way to help identify student strengths and weaknesses, misconceptions in understanding, and test for knowledge (Rittle-Johnson & Jordan, 2016). An important benefit of online assessments is that they are self-paced and that they give the student immediate feedback (Rittle-Johnson & Jordan, 2016). Konstantopoulous, Miller, and van der Ploeg (2013) studied the use of interim computer assessments with students in Grades K-8 in the state of Indiana and found that students in Grades 3-8 who worked on interim computer assessments achieved higher scores on state exams. Rittle-Johnson and Jordan (2016) examined studies on interim computer assessments and found that assessments that focus on misconceptions and accuracy are a good way to help with learning mathematics.

Online collaboration tools are yet another technology tool that can aid in the teaching and learning of mathematics. Research on instructional practices for mathematics achievement found that there is a correlation between mathematics achievement and collaboration in the mathematics classroom (Bottia, Moller, Michelson, & Stearns, 2014). One such online collaboration tool was created by The Institute of Education Science called TechPALS, where students work on mathematics problems collaboratively using devices (Roshcelle et al., 2010). Roshcelle et al. (2010) completed a study with students in Grade 4 using TechPALS and while the findings were not extensive, the results demonstrated that students who used TechPALS showed a greater knowledge of fractions and were more likely to ask questions.

Calculators are another technology tool that can be used purposefully in math to help with achievement. The Ontario Mathematics Curriculum for students in Grades 1-8 supports the use of calculators and explicitly states "the computer and the calculator should be seen as important problem-solving tools to be used for many purposes" (Ontario Ministry of Education, 2005, p. 15). The National Council of Teachers of Mathematics (2015) echoes this sentiment with their position statement that calculators should be used strategically to help with reasoning and problem-solving. Katsberg and Leatham (2005) conducted research on graphing calculators in secondary school mathematics and found that access to and use of graphing calculators can improve achievement and facilitate understanding. Harskamp, Suhre, and Van Streun (1998) conducted a study with twelve Grade 12 classes and found that students with access to graphing calculators had a better repertoire of problem-solving strategies than those who did not. Nonetheless, NCTM (2015) makes it a point to remind educators that calculators do not negate the need for students to develop proficiency in mental and paper and pencil calculations, and should be used strategically to support learning.

Interactive white boards are yet another example of technology that can be used strategically in the mathematics classroom. Using interactive white boards in mathematics as dynamic and non-dynamic presentation tools have been shown to increase student engagement and improve whole-class discussion (Bruce, 2012). Interactive whiteboards allow for a variety of representations, student interaction, and visually stimulating graphics (Bruce, 2012). However, interactive white boards are not a magic bullet for student understanding, as teachers must still focus on teaching strategies, pedagogy, and assessments to make the class effective.

**Figure 10: Teaching and Learning with Technology**

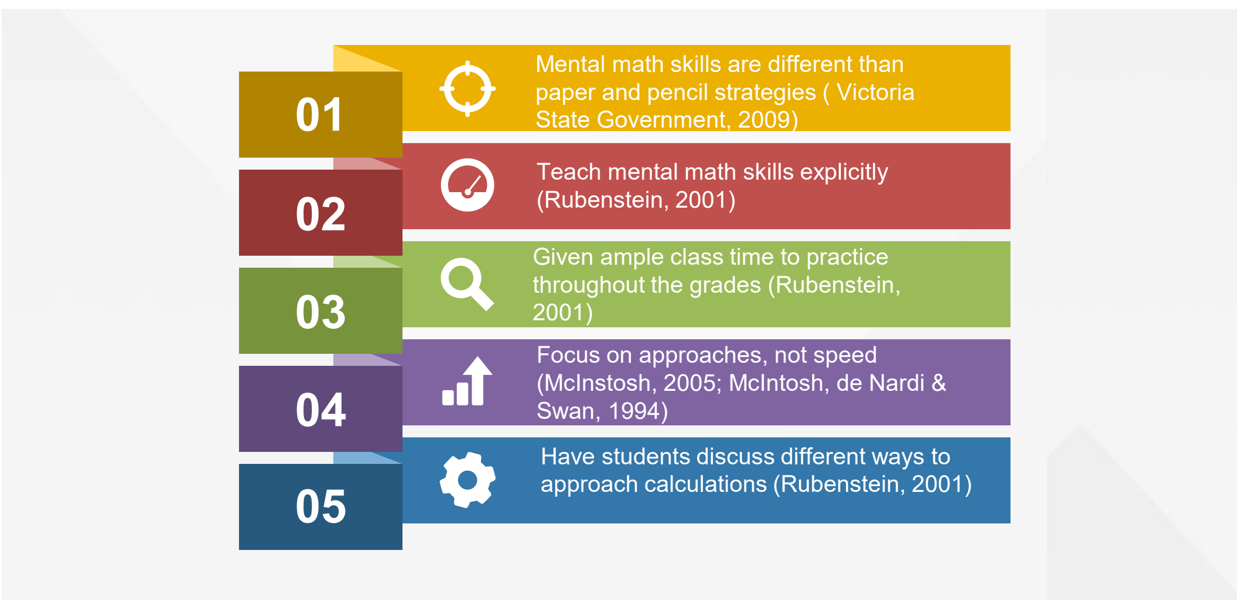


## Develop Mental Mathematics Skills

Mental math skills are important and are often used in commerce, work, and everyday activities, yet many students are calculator dependent. Mental math strategies are different than paper and pencil strategies and need to be cultivated (Victoria State Government, 2009). The Victoria State Government in Australia (2009) argues that students should be taught mental math strategies and develop these strategies at school throughout the grades. Recommendations to develop mental math skills include:

- Teaching mental math skills (Victoria State Government, 2009)
- Providing class time to practice (Rubenstein, 2001)
  - Spending 10-15 minutes a few times a week working on mental math
  - Focusing on approaches, not speed (McInstosh, 2005; McIntosh, de Nardi & Swan, 1994)
- Discussing different strategies used (McInstosh, 2005; McIntosh, de Nardi & Swan, 1994)
- Using examples in students' daily life where mental math is used and have them explain different ways to approach the calculations (Rubenstein, 2001).

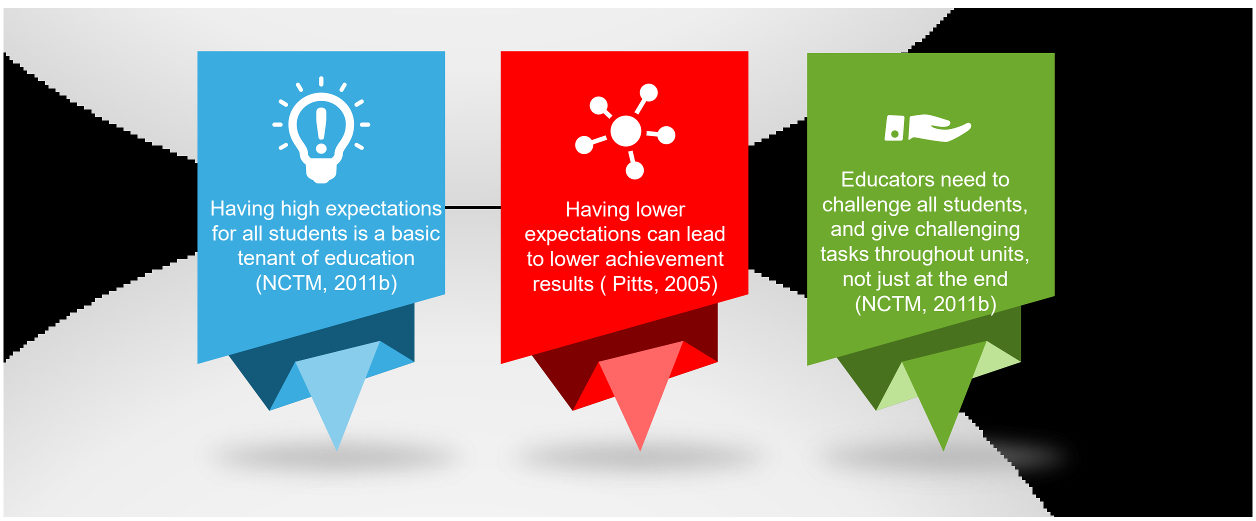
**Figure 11: Mental Mathematics Skills**



## Have High Expectations

Having high expectations of students in mathematics is at the core of good teaching (Jamar & Pitts, 2005). Having high expectations means believing all students can learn, be challenged, and push themselves as far as they can go (NCTM, 2011b). The NCTM (2011b) explains that holding high expectations is a fundamental tenet of equity since providing challenging and stimulating courses and mathematics environments should be a basic right for all students. As such, the Ontario Mathematics Curriculum Grades 1-8, maintains that "teachers should have high expectations for all students" (Ontario Ministry of Education, 2005, p. 29). Jamar and Pitts (2005) studied the effect of teacher expectations with African-American students and found that teachers who had low expectations for students can lead to lower achievement results. Barshay (2016) reports a study by the OECD found that teachers who taught students from a low socio-economic background provided less rigorous mathematics learning to their students than those teachers who taught high socio-economic students. As such, NCTM (2011b) recommends that all students should be challenged, and suggests students should be given challenging tasks throughout units of learning, not just at the end of a unit.

**Figure 12: Importance of Having High Expectations**

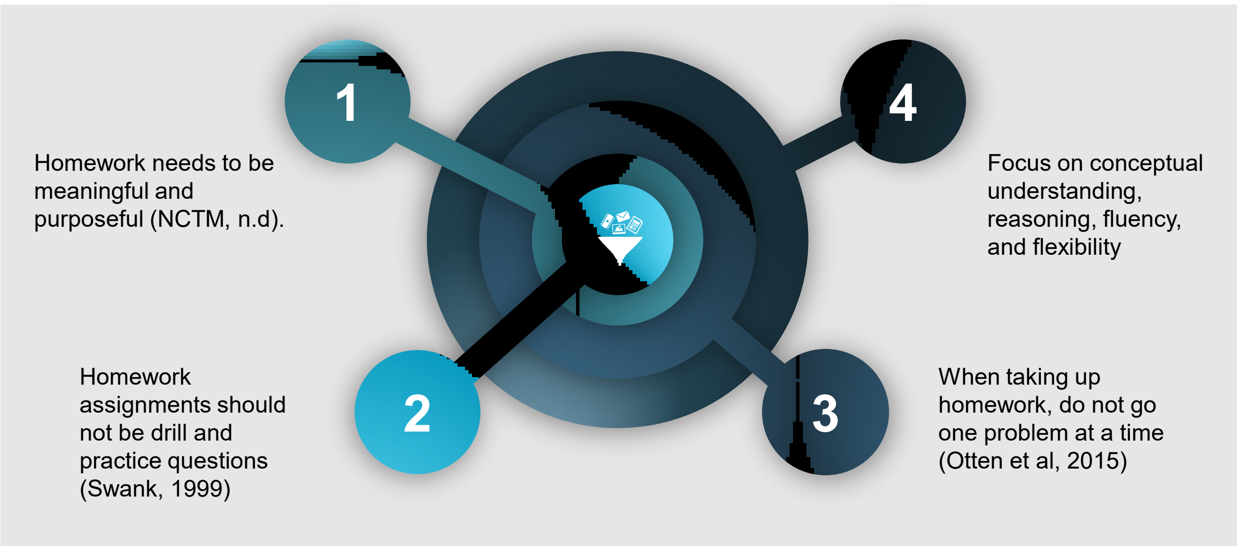


## Assign Meaningful Homework

In many mathematics classrooms, homework is assigned on a regular basis, but what does the research say about homework? According to the NCTM (n.d.), homework can improve academic achievement, but the homework needs to be meaningful and purposeful. Swank (1999) studied whether drill based homework assignments improve achievement in math, and found no significant difference between students who completed the homework assignments and those who did not. As such, Swank (1999) argues that homework assignments should not be drill and practice tasks.

Another important factor with homework is how to effectively take up homework. Otten, Cirillo, and Herbel-Eisenmann (2015) explain that often a large portion of class time is taken up with going over homework, one problem at a time, talking about correct answers or explanations. Otten et al., (2015) suggest instead of focusing on the mathematical ideas in the problems as a whole, similarities and differences between problems and actual student approaches and thinking to the problems. Overall, the focus of homework should be to improve conceptual understanding, fluency, and flexibility in math.

**Figure 13: Assign Meaningful Homework**



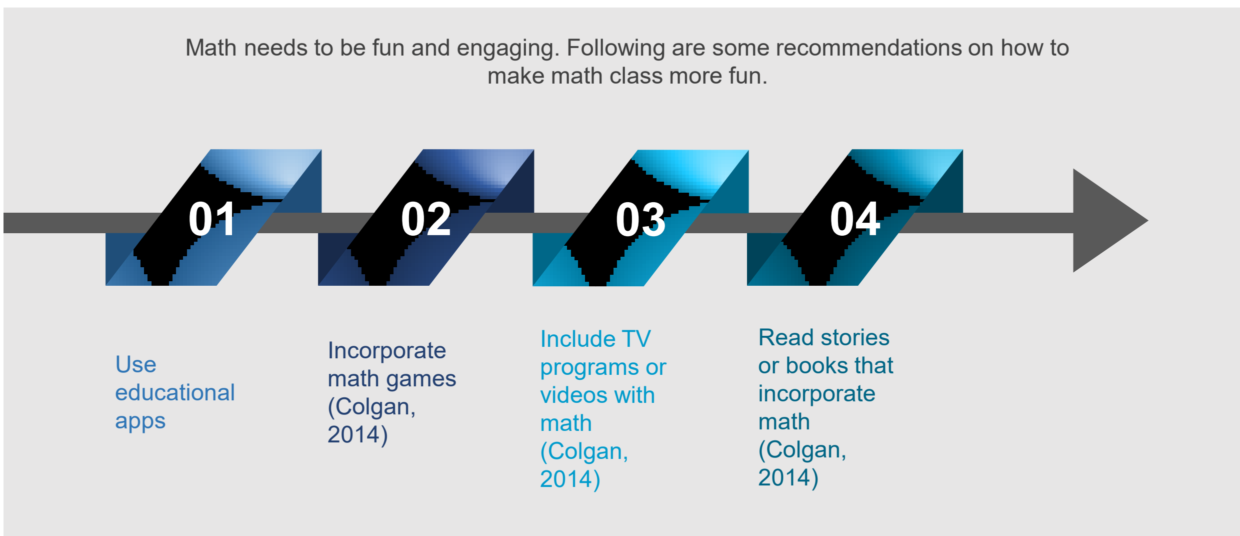
## Make Mathematics Fun

Mathematics needs to be fun and engaging. According to Colgan (2014), a large majority of students find mathematics "boring, mostly irrelevant and unrewarding" (Colgan, 2014, p. 1). This need not be the case, however, as educators should strive to use resources and strategies that capture student interest and spike motivation (Colgan, 2014).

Examples include use of:

- Educational apps (such as Kahoot!)
- Math games (such as Mathemagic or the Prime Radicals Game) (Colgan, 2014)
- TV programs/videos (such as the TVOkids program The Prime Radicals) (Colgan, 2014)
- Stories and books that incorporate mathematics (such as Mathemagic! Number Tricks) (Colgan, 2014)
- Use physical movements in the classroom to act the mathematics (Rittle-Johnson & Jordan, 2016)
- 'Math busking', using street performing to learn about math (Robertson Program for Inquiry-based Teaching in Mathematics and Science, 2016)

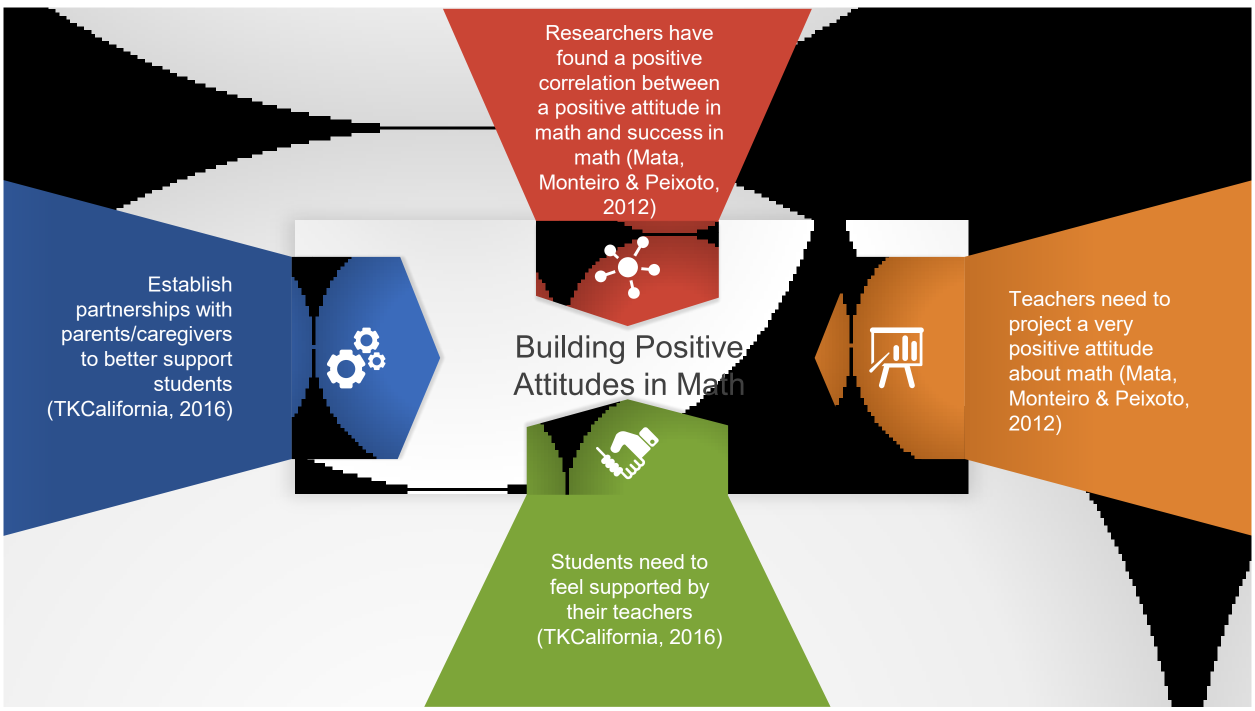
**Figure 14: Making Mathematics Fun**



## Building Positive Attitudes in Mathematics

Having a positive outlook in mathematics is an important factor in student achievement (Colgan, 2014). Several researchers have found a positive correlation between positive mathematics attitudes and mathematics achievement, demonstrating that students with a positive attitude about mathematics had greater success in mathematics (Mata, Monteiro, & Peixoto, 2012). Unfortunately, many students have negative attitudes towards mathematics and feel that they are not good at math (Colgan, 2014). In the most recent TIMMS study conducted in 2011, only 35% of students in Grade 4 and 32% of students in Grade 8 reported liking mathematics (EQAO, 2011). Several factors in the school environment including teacher support, student-to student interaction, and teacher expectations can affect attitudes in mathematics (Mata et al., 2012). Akey (2006) concluded that students have a better attitude towards mathematics when their teacher has a very positive attitude about mathematics (Mata et al., 2012). Additionally, teachers with a more positive attitude in mathematics are perceived to be better at supporting students (Mata et al., 2012). Furthermore, teachers who establish partnerships with parents/caregivers are better able to support students (TKCalifornia, 2016).

**Figure 15: Building Positive Attitudes in Mathematics**

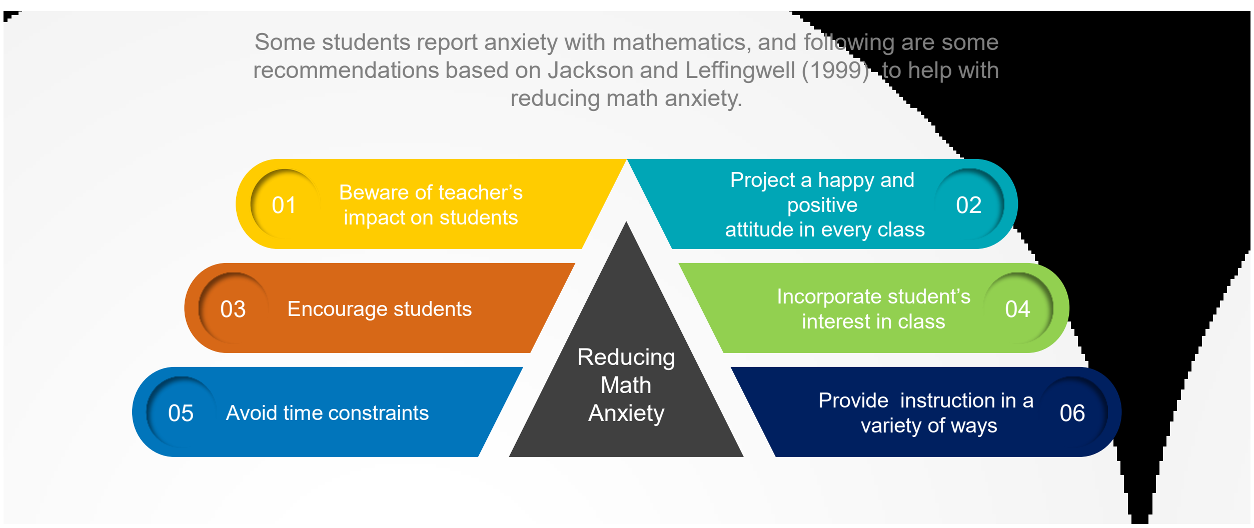


## Reduce Mathematics Anxiety

Some students report anxiety or nervousness in mathematics, and as educators, we must be able to recognize that math anxiety is real, and be aware of strategies to reduce mathematics anxiety with our students (Stuart, 2000). Mathematics anxiety can be characterized by math avoidance, obsessing over everything that is not understood, or viewing math as a punishment (Stuart, 2000). Most often a lack of confidence in mathematical abilities can lead to anxiety in mathematics, but many other things can attribute to mathematics anxiety including teacher attitudes, peer attitudes, and family attitudes (Stuart, 2000). Ho et al. (2000) conducted a study with 671 students in Grade 6 in China, Taiwan, and the United States and found that math anxiety has an impact on student achievement. As such, teachers must be aware of teaching strategies to help reduce math anxiety. Jackson and Leffingwell (1999) conducted a study with 157 pre-service elementary school teachers on their personal experience with math anxiety in the classroom and based on their findings, the authors provided the following recommendations for teachers to help with math anxiety:

- Be aware of teacher impact on students
- Project a happy and positive attitude that you want to teach in each and every one in your classes
- Be encouraging. Do not make students feel embarrassed for asking for help
- Try to include student interests into the class
- Provide extra time or help for students who may suffer from math anxiety
- Give instructions both in written and verbal formats (Jackson & Leffingwell, 1999)

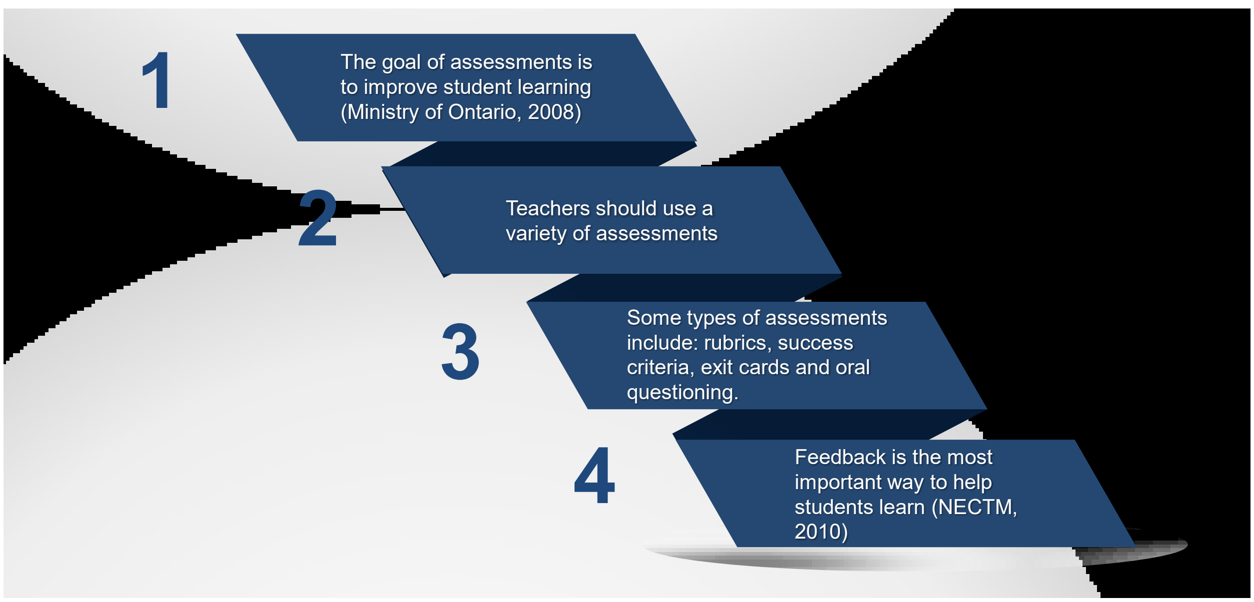
**Figure 16: Recommendations to Reduce Mathematics Anxiety**



## Provide Varied and Ongoing Assessment

The Ontario Mathematics Curriculum Grades 1-8 explains the goal of assessments is to improve student learning (Ontario Ministry of Education, 2005). Assessments should be varied and ongoing, addressing "what students learn and how well they learn it" (Ontario Ministry of Education 2005, p. 18). Different kinds of mathematics assessments include: "assignments, day to day observations, conversations/conferences, demonstrations, projects, performances and tests" (Ontario Ministry of Education 2005, p. 18). Students should have opportunities to do more than just tests and quizzes in mathematics. Teachers should use varied types of assessments to improve student learning and reinforce that mathematics is more than just right and wrong answers. Using rubrics, success criteria, exit cards, journals, and oral questioning, teachers can motivate students to learn and provide more immediate and tailored feedback. Feedback is one of the most important ways we can motivate students to learn mathematics (National Centre for Excellence in the Teaching of Mathematics [NECTM], 2010). Feedback helps students track their progress, fix mistakes, and extend learning (NECTM, 2010).

**Figure 17: Mathematics Assessment**

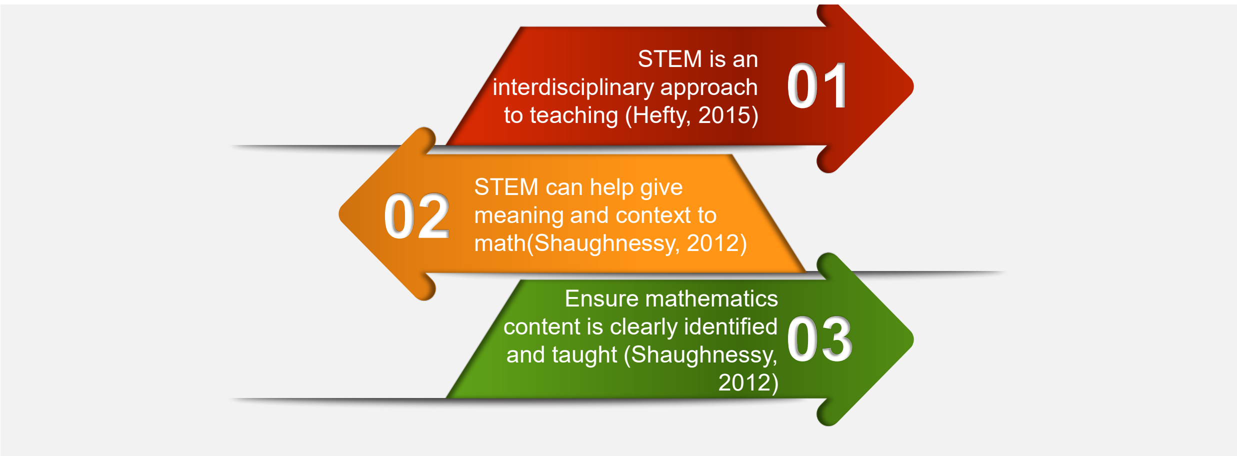



## Give Meaning to Mathematics through Science, Technology, Engineering and Mathematics

Science, Technology, Engineering and Mathematics (STEM), education is an interdisciplinary approach to teaching that allows students to create, build, understand, and make connections to the world around them in an authentic way (Hefty, 2015). Shaughnessy (2012) explains that STEM is becoming a driving force in educational development and gives students a way to use mathematics in meaningful ways. Robotics is an example of a STEM activity that can be designed to develop knowledge of math such as proportional reasoning (Silk, Higashi, Shoop, & Schunn, 2010).

While STEM can give meaning to mathematics to students, there are some important considerations about mathematics learning that need to be discussed. Science, Technology, Engineering and Mathematics challenges can help develop students' problem-solving and critical thinking skills, but some are concerned that STEM challenges dilute the mathematics and as such, students may not be learning the required curriculum in mathematics. Shaughnessy (2012) suggests that the mathematics content needs to be clearly identified, taught, and discussed when working on STEM activities.

**Figure 18: Give Meaning to Mathematics through STEM**

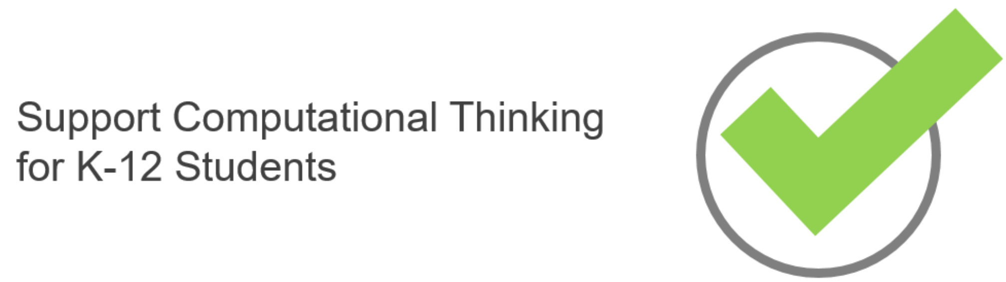



### Support Computational Thinking

Computational thinking is about "solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science" (Wing, 2006, p. 1). Wing (2006) argues computational thinking should be a fundamental part of the curriculum. In the United States, Next Generation Science Standards computational thinking is one of the core aspects of the Science and Engineering Practices for K-12 (National Science Teachers Association [NSTA], 2013). Students are expected to "engage in computational thinking, which involves strategies for organizing and searching data, creating sequences of steps called algorithms, and using and developing new simulations of natural and designed systems" (NSTA, 2013, p. 1). In Ontario, one of the seven mathematical processes are computational strategies, which focuses on students collecting, organizing, and sorting data (Ontario Ministry of Education, 2005), but does not seem to have much of a focus on computational thinking in relation to computer science[3]. The National Council of Teachers of Mathematics President, Matt Larson, argues that mathematics educators should support efforts for computational thinking, but to be cautious there still remains a strong emphasis on mathematical thinking and learning in our classrooms (Larson, 2016).

[3] This is based on observations of the authors.

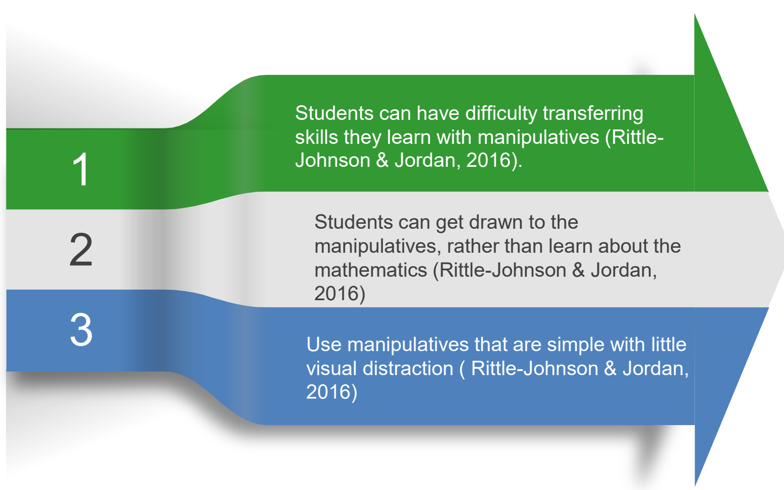
**Figure 19: Computational Thinking**




### Use Manipulatives with Minimal Visual Distractions

Manipulatives are very popular tools in the mathematics classroom, with over 90% of Kindergarten teachers using manipulatives (Rittle-Johnson & Jordan, 2016). However, in July 2016 the Institute of Education Science (IES) at the U.S. Department of Education released a synthesis of mathematics education research from 2002-2013 explaining that multiple studies have reported manipulatives having little or negative effects on learning (Rittle-Johnson & Jordan, 2016). “IES-funded experimental research highlights the difficulty students have transferring skills they learn with manipulatives to solve problems without manipulatives (as is typical on tests)” (Rittle-Johnson & Jordan, 2016, p. 12). In some cases, students get drawn to the materials of manipulatives, rather than the abstracts of mathematics behind them (Rittle-Johnson & Jordan, 2016). For example, McNeil, Uttal, Jarvin, and Sternberg (2009) studied students in Grades 4-6 who used manipulatives when working on problems about money and found that students made more mistakes when the money looked realistic than when they were plain. As such, the IES recommends that manipulatives used in the classroom should be simple with minimal visual distractions (Rittle-Johnson & Jordan, 2016).

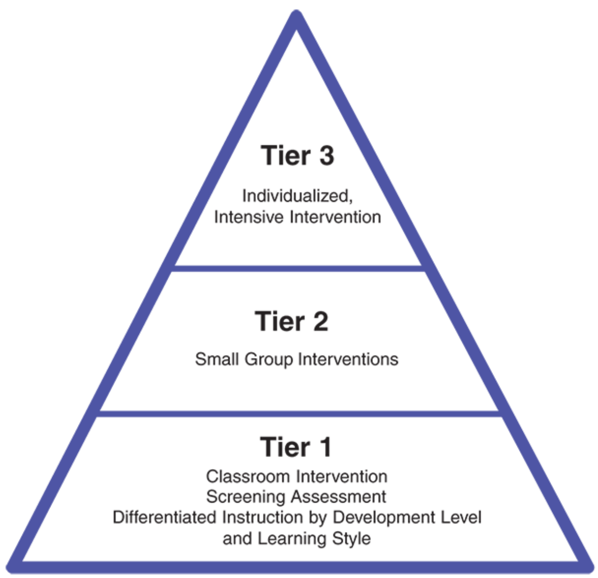
**Figure 20: Information on Manipulatives**



### How to Implement Successful System Wide Interventions

In keeping with systems that are in place in areas such as special education, The Office of Superintendent of Public Instruction (2011) has recently adopted the "Response to Intervention (RTI) [which] is a framework used to increase student achievement and prevent academic failure by addressing the needs of all students" (p. 1). The RTI is a "well-defined multi-level instructional system that includes a core program for all students" (p. 1). The RTI is "based on early identification of students at risk of poor performance through screening, the student receives increasingly intensive levels of intervention designed to help the student become successful" (p. 1). The following image displays the intervention mode which can be used universally in education and is not limited to math.

**Figure 21: Response to Intervention**




**Source:** The Office of Superintendent of Public Instruction, 2011.

Response to Intervention (RTI) involves repetition of high-quality instruction/intervention harmonized with student needs; and uses learning over time and level of performance to make important educational decisions to guide instruction (Gresham & Little, 2012). Response to Intervention is actually a sequence of assessing, intervening, and reasoning (Robins & Antrim, 2013). "The key to success is scheduling of intervention efforts to supplement, but not supplant, core instruction" (Fisher & Frey, 2013, p. 112).

A significant element is the use of data (collected continuously) based decision making focused on instruction intensity students require to meet learning needs (Pool, Carter, Johnson, & Carter, 2013). Specifically, **Tier 1** (whole class instruction) is made up of instructional practices a teacher uses with all students and includes:

1. The core instructional program along with
2. classroom routines that provide opportunities for instructional differentiation,

3. accommodations that permit access to the primary prevention program for all students, and,
4. problem-solving strategies designed to address students' motivational problems that interfere with them performing the academic skills they possess. (Fuchs et al., 2011, pp. 372-373)

Ongoing, timely assessment and evaluation unfolds and is used to re-examine student progress and status regularly. Within **Tier 2** (teacher small-group instruction) we find the,

1. use of screening data and criteria for placement,
2. interventions based on strong instructional design principles, and
3. frequent progress monitoring. (NCRTI, 2010)

Within **Tier 3** teachers provide concentrated, individual, and/or small group instruction for students who are below grade level even with Tier 2 supports (Dobbins et al., 2014). "The whole school has to be involved with the effort for it to be successful" (Fisher & Frey, 2013, p. 110) and all teachers should have professional development that illuminates and builds the skills needed to provide interventions for students; as all should know what instruction and learning looks like at all Tier levels (Fisher & Frey, 2013).

## SUMMARY AND RECOMMENDATIONS

This literature review discussed research-based instructional strategies in teaching mathematics that can help support and foster student development in mathematics. Mathematics classes need to promote problem-solving, creativity, collaboration, investigating and fun through inquiry-based learning, varied teaching strategies, and ongoing and changing assessment methods. Table 1 summarizes the main research-based recommendations.

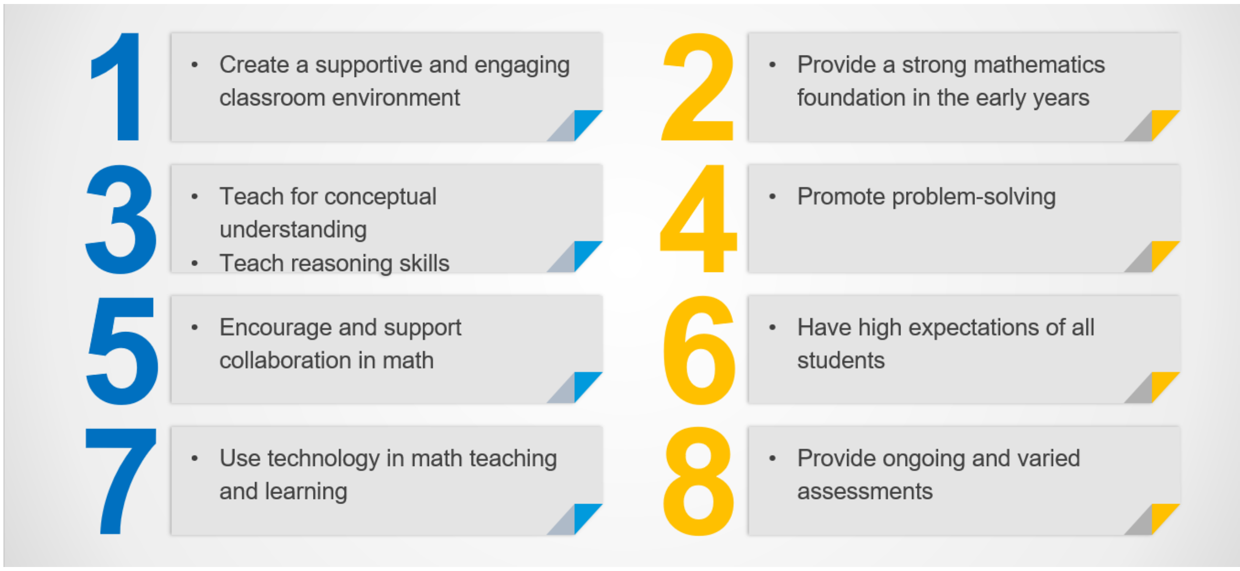
**Table 1: Summary of Recommendations for Teaching and Learning Mathematics**

| Evidence-based Recommendations | Details |
|---|---|
| Creating a Supportive and Engaging Classroom Environment Via Response to Intervention (RTI) | • Make the class a place of investigation (Suurtamm et al., 2015)<br>• Support unusual ideas and responses (Feldhusen & Treffinger (1985); Nickerson (1999); Sternberg & Williams (1996)<br>• Make the classroom feel like a community (Bruce, 2007)<br>• Encourage classroom dialogue through Math Talk (Hufferd-Ackles et al., 2004) |
| Providing a Strong Mathematics Foundation in the Early Years | • A strong mathematics foundation can play an important role in future academic achievement (Duncan, 2007)<br>• All children should have access to evidence-based quality early years' programs (National Council of Teachers of Mathematics [NCTM], 2013)<br>• There needs to be rigorous standards based curriculum for the early years (NCTM, 2013)<br>• Teachers need to be knowledgeable about child development (Student Achievement Division, 2011), and classroom pedagogy for young learners (Ontario Ministry of Education, 2003),<br>• Maintain a positive attitude about mathematics (Ontario Ministry of Education, 2003), and be knowledgeable about mathematics (Student Achievement Division, 2011) |
| Teaching for Conceptual Understanding | • Stress conceptual understanding over procedural understanding (Lawson, 2007)<br>• Students should be taught the concepts behind procedures either before or during the instruction of procedures, not after (NCTM, n.d.) |
| Teaching Reasoning Skills | • Teach reasoning (such things as identifying similarities/differences, size, or shape) as part of the mathematics classroom (Rittle-Johnson & Jordan, 2016) |

| Table 1: Summary of Recommendations for Teaching and Learning Mathematics | |
|---|---|
| **Evidence-based Recommendations** | **Details** |
| Promoting Problem-solving | • Present complex and rich problems allowing for multiple entry points, different approaches, scaffolding, and engagement without imposed procedural steps (Suurtamm, Quigley, & Lazarus, 2015)<br>• Have students solve open-ended problems (Barwell, 2011; Hoffman & Brahier, 2008; Suurtamm et al., 2015)<br>• Allow students to create and solve their own problems (Barwell, 2011)<br>• Allow students to work on problems mirroring authentic real life scenarios (Archbald & Newmann, 1988)<br>• Allow students to compare multiple ways to solve problems (Rittle-Johnson & Jordan, 2016)<br>• Allow students to critique common mistakes (Rittle-Johnson & Jordan, 2016)<br>• Allow students to mathematize the world around them (Barwell, 2011) |
| Creativity in Mathematics | • Allow time for students to be creative (Burns, 1992)<br>• Give opportunities to students to work on problems that require resourcefulness and creativity (Pehkonen, 1997) |
| Encouraging and Supporting Collaboration in Mathematics | • Mathematics collaboration should be encouraged and supported in the classroom (MacMath et al., 2009)<br>• Use technology to support student collaboration in math (Rittle-Johnson & Jordan, 2016) |
| Providing an Inquiry Environment | • Provide a learning environment where students are active learners and where students can take on the role of a mathematician (PRIMAS, 2011) |
| Use Three-part Mathematics Lesson Plans | • Three-part math lessons are a lesson planning technique focused on inquiry-based learning supported by NCTM and the Ontario College of Teachers to teach mathematics standards and create an engaging classroom |
| Using Technology in Teaching and Learning Mathematics | • Online assessments are one technology tool that can be useful in teaching and learning mathematics (Rittle-Johnson & Jordan, 2016). Use online assessments that show students their misconceptions, not just correct answers.<br>• Use online collaboration tools (Roshcelle et al., 2010)<br>• Use calculators strategically (NCTM, 2015)<br>• Using interactive white boards to increase student engagement and ignite whole-class discussion (Bruce, 2012). |
| Developing Mental Mathematics Skills | • Teach mental math computation. It is different than paper and pencil procedures (Victoria State Government, 2009)<br>• Give students class time to work on mental math and discuss different approaches (Rubenstein, 2001) |

| Table 1: Summary of Recommendations for Teaching and Learning Mathematics | |
|---|---|
| **Evidence-based Recommendations** | **Details** |
| Having High Expectations | • Teachers should have high expectations in mathematics for all students and provide challenging and stimulating courses and classroom environments (NCTM, 2011a) |
| Make Mathematics Fun | • Many students find math boring and disengaging. Use games, apps, TV programs, and books to make math fun (Colgan, 2014)<br>• Use physical movements in the classroom (Rittle-Johnson & Jordan, 2016) |
| Building Positive Attitudes in Mathematics | • Teachers need to project a positive attitude in mathematics<br>• Teachers need to support students and establish partnerships with the students' parents/caregivers to better support students (TKCalifornia, 2016) |
| Providing Varied and Ongoing Assessment | • Teachers should use a variety of assessments such as observations, conversations, demonstrations, projects, journals, problems, and tests to improve student learning (Ontario Ministry of Education, 2005)<br>• The classroom should include more than tests and quizzes.<br>• Teachers should give students ongoing feedback when learning mathematics (National Centre for Excellence in the Teaching of Mathematics [NECTM], 2010) |
| Giving Meaning to Mathematics with STEM | • STEM activities allow students to make mathematics meaningful by making connections and critically think about the world around them (Hefty, 2015).<br>• The mathematics in the STEM activity needs to be identified, taught, and discussed (Shaughnessy, 2012) |
| Support Computational Thinking | • Mathematics educators should support efforts for computational thinking, but to be cautious as there still remains a strong emphasis on mathematical thinking and learning in our classrooms (Larson, 2016). |
| Use Manipulatives with Minimal Visual Distractions | • Research has shown that students can get distracted with manipulatives that are too realistic or visually stimulating. Use manipulatives with minimal visual distractions (Rittle-Johnson & Jordan, 2016) |

**Figure 22: Summary of Key Recommendations**




It is our hope that these instructional practices will be incorporated into mathematics classrooms to improve our students' mathematical abilities and attitudes in mathematics. The incorporation of these instructional practices can be fostered through system-wide professional learning.

## REFERENCES


**Akey, T. M. (2006).** School context, student attitudes and behavior, and academic achievement: An exploratory analysis. *MDRC*. Retrieved from http://www.mdrc.org/sites/default/files/full_519.pdf

**Archbald, D. A., & Newmann, F. M. (1988).** *Beyond standardized testing: Assessing authentic academic achievement in the secondary school*. Reston, VA: National Association of Secondary School Principals. (ED301587). Retrieved from http://eric.ed.gov/?id=ED301587

**Balfanz, R. (1999).** Why do we teach young children so little mathematics? Some historical considerations. In J. V. Copley (Ed.), *Mathematics in the early years.* (pp. 3-10). Reston, VA: National Council of Teachers of Mathematics.

**Ball, D. (2008).** Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5). 389–407.

**Barshay, J. (2016).** Is it better to teacher pure math instead of applied math? The Hechinger Report. Retrieved from http://hechingerreport.org/pure-math-better-applied-math/

**Barwell, R. (2011).** Word problems: Connecting language, mathematics and life what works? Research into Practice: Research Monograph 34 . Literacy and Numeracy Secretariat. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_Word_Problems.pdf

**Beckmann, C. E., Thompson, D. R., & Rubenstein, R. N. (2009).** *Teaching and learning high school mathematics*. Hoboken, NJ: John Wiley & Sons.

**Beigie, D. (2008).** Integrating content to create problem-solving opportunities. *Mathematics Teaching in the Middle School*, *13*(6), 352–360.

**Berger, I. (n.d).** Rethinking math. *Research in practice: Mathematical thinking in the early years*. Retrieved from http://earlychildhood.educ.ubc.ca/community/research-practice-mathematical-thinking-early-years

**Bottia, M.C., Moller, S., Mickelson, R.A., and Stearns, E. (2014).** Foundations of mathematics achievement: Instructional practices and diverse kindergarten students. *The Elementary School Journal, 115*(1): 124-150.

**Bruce, C.D. (2007).** *Student interaction in the math classroom: Stealing ideas or building understanding*. What Works? Research into Practice. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Bruce.pdf

**Bruce, C.D. (2012).** *Technology in the mathematics classroom*. What Works? Research into Practice: Research Monograph 38. Literacy and Numeracy Secretariat. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_technology.pdf

**Burns, M. (1992).** *About teaching mathematics: A K-12 resource*. Cuisenaire Company of America.

**Canadian Child Care Federation. (2010).** *Foundations for numeracy: An evidence-based toolkit for early learning practitioners*. Canadian Language and Literacy Research Network. Retrieved from http://eyeonkids.ca/docs/files/foundations_for_numeracy.pdf

**Colgan, L. (2014).** *Making math children will love: Building positive mathitudes to improve student achievement in mathematics.* What Works? Research into Practice Research Monograph 56. Student Achievement Division, Ontario Ministry of Education. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_MakingMath.pdf

**Dobbins, A., Gagnon, J. C., & Ulrich, T. (2014).** Teaching geometry to students with math difficulties using graduated and peer-mediated instruction in a response-to-intervention model. *Preventing School Failure*, *58*(1), 17-25.

**Dooley, T., Dunphy, T., Shiel, G., Butler, D., Corcoran, D., Farrell, T., NicMhuirí, S., O'Connor, M., & Travers, J. (2014).** *Mathematics in early childhood and primary education (3-8 years): Teaching and learning* (No. 18). National Council for Curriculum and Assessment (NCCA) Research Report.

**Dossey, J. A., McCrone, S., & O'Sullivan, C. (2006).** *Problem-solving in the PISA and TIMSS assessment 2003*. U.S. Department of Education.

**Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Japel, C. (2007).** School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446.

**EQAO. (2011).** *Trends in International Mathematics and Science Study (TIMSS), 2011, Ontario Report.*

**EQAO. (2012).** *Programme for International Student Assessment (PISA), 2012, Highlights of Ontario Student Results*.

**Evans, C. R., & Dion, K. L. (1991).** Group cohesion and performance: A meta-Analysis. *Small Group Research*, *22*(2), 175–186. doi: 10.1177/1046496491222002

**Feldhusen, J. F., & Treffinger, D. J. (1985).** *Creative thinking and problem solving in gifted education*. Dubuque, IA: Kendall/Hunt Publishing Company.

**Fleming, G. P. (2000).** *The effects of brainstorming on subsequent problem-solving*. Unpublished doctoral dissertation. Ann Arbor, MI: Graduate School of Saint Louis University.

**Fisher, D., & Frey, N. (2013).** Implementing RTI in a high school: A case study. *Journal of Learning Disabilities, 46*(2), 99-114.

**Foote, M., Earnest, D., Mukhopadhyay, S., & Curcio, F. (2014).** *Implementing the CCSSM through problem solving, grades 3-5*. Reston, VA: National Council of Teachers of Mathematics.

**Fuchs, L. S., Compton, D. L., Fuchs, D., Hollenbeck, K. N., Hamlett, C. L., & Seethaler, P. M. (2011).** Two-stage screening for math problem-solving difficulty using dynamic assessment of algebraic learning. *Journal of Learning Disabilities, 44*(4), 372-380.

**Gadanidis, G., & Geiger, V. (2010).** A social perspective on technology enhanced mathematical learning - from collaboration to performance. *ZDM*, 42(1), 91–104.

**Gifford, S. (2015).** *Early years mathematics: How to create a nation of mathematics lovers?* Retrieved from https://nrich.maths.org/11441

**Greer, B. (1997).** Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction*, *7*(4), 293-307.

**Gresham, G., & Little, M. (2012).** RtI in math class.*Teaching Children Mathematics 19*(1), 20-29.
**Guilford, J. P. (1959).** Traits of creativity. In H. H. Anderson (Ed.). *Creativity and its Cultivation* (pp. 142-161). New York, NY: Harper and Brothers.

**Hasler-Waters, L., & Napier, W. (2002).** Building and supporting student team collaboration in the virtual classroom. *Quarterly Review of Distance Education*, *3*(3), 345-52.

**Harskamp, E. G., Suhre, C. J. M., & van Struen, A. (1998).** The graphics calculator in mathematics education: An experiment in the Netherlands. *Hiroshima Journal of Mathematics Education, 6*, 13-31.

**Hefty, L. J. (2015).** STEM Gives Meaning to Mathematics. *Teaching Children Mathematics*, *21*(7), 422-429.

**Hiebert, J. (1999).** Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30*(1), 3–19.

**Ho, H. Z., Senturk, D., Lam, A. G., Zimmer, J. M., Hong, S., Okamoto, Y., Chiu, S. Y., C., Nakazawa, Y., & Wang, C. P. (2000).** The affective and cognitive dimensions of math anxiety: A cross-national study. *Journal for Research in Mathematics Education*, 31(3), 362-379.

**Hoffman, L. R., & Brahier, D. J. (2008).** Improving the planning and teaching of mathematics by reflecting on research. *Mathematics Teaching in the Middle School*, *13*(7), 412–417.

**Hufferd-Ackles, K., Fuson, K. C. & Gamoran Sherin, M. (2004).** Describing levels and components of a Math-Talk Learning Community. *Journal for Research in Mathematics Education,* 35 (2): 81-116.

**Jackson, C. D., & Leffingwell, R. J. (1999).** The role of instructors in creating math anxiety in students from kindergarten through college. *The Mathematics Teacher*, *92*(7), 583-586.

**Jamar, I., & Pitts, V. R. (2005).** High expectations: A "how" of achieving equitable mathematics classrooms. *Negro Educational Review*, *56*(2/3), 127.

**Kastberg, S., & Leatham, K. (2005).** Research on graphing calculators at the secondary level: Implications for mathematics teacher education. *Contemporary Issues in Technology and Teacher Education*, *5*(1), 25-37.

**Kidd, J.K., Carlson, A.G., Gadzichowski, K.M., Boyer, C.E., Gallington, D.A., and Pasnak, R. (2013).** Effects of Patterning Instruction on the Academic Achievement of 1st-Grade Children. *Journal of Research in Childhood Education*, 27(2): 224-238 Konstantopoulos, S., Miller, S., and van der Ploeg, A. (2013). The Impact of Indiana's system of interim assessments on mathematics and reading achievement. *Educational Evaluation and Policy Analysis*, *35*(4), 481-499.

**Larson, M. (2016).** *Computer science and mathematics education*. President's Messages.

**Lawson, A. (2007).** *Learning mathematics vs. following "Rules": The value of student-generated methods*. What Works? Research into Practice. Student Achievement Division.

**Lawson, A. (2016).** *The mathematical territory between direct modelling and proficiency*. What Works? Research into Practice. Student Achievement Division.

**Lewin, K. (1943).** Forces behind food habit and methods of changes. In: *The problem of changing food habits: Report of the Committee on Food Habits*. Washington, D. C. National Academy of Sciences, p. 35-65. Bulletin of National Research Council 108.

**MacMath, S., Wallace, J. & Xiaohong, C. (2009).** *Problem-based learning in Mathematics- A tool for developing students' conceptual knowledge*. What Works? Research into Practice.  Research Monograph #22. The Literacy and Numeracy Secretariat.

**Mata, M. D. L., Monteiro, V., & Peixoto, F. (2012).** Attitudes towards mathematics: Effects of individual, motivational, and social support factors. *Child Development Research*, *2012*.

**McNeil, N., Uttal, D.H., Jarvin, L., and Sternberg, R. J. (2009).** Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction, 19*(2): 171-184.

**McIntosh, A. & Dole, S. (2005).** *Mental computation: A strategies approach*. Hobart: Department of Education, Tasmania.

**McIntosh, A., De Nardi, E., & Swan, P. (1994).** *Think mathematically! How to teach mental maths in the primary classroom*. Melbourne, AU: Longman Cheshire.

**Meiers, M., Reid, K., McKenzie, P., & Mellor, S. (2013).** *Literacy and numeracy interventions in the early years of schooling: A literature review*. Report to the Ministerial Advisory Group on Literacy and Numeracy. Australian Council for Educational Research. Retrieved from http://research.acer.edu.au/policy_analysis_misc/20/

**Ontario Ministry of Education. (2003).** *Early math strategy*. The Report of the Expert Panel on Early Math in Ontario. Retrieved from http://www.edu.gov.on.ca/eng/document/reports/math/math.pdf

**Ontario Ministry of Education. (2005).** *The Ontario curriculum grades 1-8 mathematics*. Retrieved from http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf

**National Mathematics Advisory Panel. (2008).** *Foundations for success: The final report of the national mathematics advisory panel*. Washington, DC: U.S. Department of Education. Retrieved from http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

**National Association for the Education of Young Children & National Association of Early Childhood Specialists in State Departments of Education. (2003).** *Early childhood curriculum, assessment, and program evaluation: Building an effective, accountable system in programs for children birth through age 8*. Joint Position Statement. Washington, DC: NAEYC.

**National Association for the Education of Young Children. (2009).** *Developmentally appropriate practice in early childhood programs serving children from birth through age 8*. Position Statement. Washington, DC: NAEYC.

**National Centre for Excellence in the Teaching of Mathematics . (2010).** *Assessment for learning in mathematics*. Retrieved from https://www.ncetm.org.uk/public/files/725964/Assessment+for+Learning+in+Mathematics.pdf

**National Center on Response to Intervention. (2010).** *Essential components of RTI - A closer look at response to intervention*. Washington, DC: U.S. Department of Education, Office of Special Education Programs, National Center on Response to Intervention.

**National Council of Teachers of Mathematics. (n.d.).** *What does research say about the effectiveness of homework?* Retrieved from http://www.nctm.org/Research-and-Advocacy/research-brief-and-clips/Homework_-What-Research-Says/

**National Council of Teachers of Mathematics. (n.d.).** *What is procedural fluency, and how do we help students develop it? A position of the National Council of Teachers of Mathematics.* Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/

**National Council of Teachers of Mathematics . (1995).** Position statement on interdisciplinary learning: Pre-K4. *Teaching Children Mathematics, 1*(6), 386-387.

**National Council of Teachers of Mathematics. (2011a).** A position of the National Council of Teachers of Mathematics. *What is the role of technology in the teaching and learning of mathematics?* Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Strategic-Use-of-Technology-in-Teaching-and-Learning-Mathematics/

**National Council of Teachers of Mathematics. (2011b).** *What does it mean to hold high expectation for students in mathematics education?* A Position of the National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/High-Expectations/

**National Council of Teachers of Mathematics. (2013).** *Why is mathematics important for early childhood learners?* A Position of the National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

**National Council of Teachers of Mathematics. (2015).** *What is the role of calculators in the elementary grades?* A Position of the National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Calculator-Use-in-Elementary-Grades/

**National Research Council. (2009).** *Mathematics learning in early childhood: Paths toward excellence and equity*. Committee on Early Childhood Mathematics, C. Cross, T. Woods, & H. Schweingruber (Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

**National Science Teachers Association [NSTA]. (2013).** *Science & engineering practices. Next generation science standards.* Retrieved from: http://nstahosted.org/pdfs/ngss/MatrixOfScienceAndEngineeringPractices.pdf

**Nickerson, R. S. (1999).** How we know - and sometimes misjudge - what others know: Imputing one's own knowledge to others. *Psychological Bulletin, 125*(6), 737-759. Retrieved from http://doi.org/10.1037/0033-

**Nikitina, S., & Mansilla, V. B. (2003).** Three strategies for interdisciplinary math and science teaching: A case of the Illinois Mathematics and Science Academy. *Goodwork Project Report Series*, *21*.

**Office of Superintendent of Public Instruction. (2011).** *Mathematics Systems Improvement Framework. An implementation guide for district leaders*. Olympia, WA: Office of Superintendent of Public Instruction. Retrieved from http://www.k12.wa.us/mathematics/pubdocs/MathImprovementFramework.pdf

**Ontario College of Teachers (n.d).** *Sketch of a three-part lesson*. Retrieved from http://professionallyspeaking.oct.ca/march_2010/features/lesson_study/three-part.aspx

**Otten, S., Cirillo, M., & Herbel-Eisenmann, B. A. (2015).** Making the most of going over homework. *Mathematics Teaching in the Middle School*, *21*(2), 98-105.

**Pehkonen, E. (1997).** The state-of-art in mathematical creativity. *ZDM*, *29*(3), 63-67.

**Polya, G. (1945).** *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

**Pool, J. L., Carter, G. M., Johnson, E. S., & Carter, D. R. (2013).** The use and effectiveness of a targeted math intervention for third graders. *Intervention in School and Clinic, 48*(4), 210-217.

**PRIMAS. (2011).** *The PRIMAS project: Promoting inquiry-based learning (IBL) in mathematics and science education across Europe*. Retrieved from http://www.primas-project.eu/servlet/supportBinaryFiles?referenceId=2&supportId=1300

**Prince Edward Island Department of Education. (2012).** *Early numeracy intervention program English pilot site report 2010-2011. Prince Edward Island Canada Education and Early Childhood Development English Programs*. Retrieved from http://www.gov.pe.ca/photos/original/eecd_ENIP201011.pdf

**Protheroe, N. (2007).** What does good math instruction look like? *Principal*, *87*(1), 51-54.

**Rittle-Johnson, B., & Jordan, N. (2016).** Synthesis of IES-funded research on mathematics: 2002-2013. *National Center for Special Education Research*. Retrieved from http://ies.ed.gov/ncer/pubs/20162003/pdf/20162003.pdf

**Rittle-Johnson, B., & Schneider, M. (2014).** Developing conceptual and procedural knowledge of mathematics. *Oxford handbook of numerical cognition.* Oxford, UK: Oxford University Press.

**Robertson Program for Inquiry-based Teaching in Mathematics and Science. (2016).** *Math busking*. Retrieved from http://www.oise.utoronto.ca/robertson/1/Math_Busking/index.html

**Roschelle, J., Rafanan, K., Bhanot, R., Estrella, G., Penuel, W.R., Nussbaum, M., & Claro, S. (2010).** Scaffolding group explanation and feedback with handheld technology: Impact on students' mathematics learning. *Educational Technology Research and Development, 58*(4), 399-419.

**Rubenstein, R. N. (2001).** Mental mathematics beyond the middle school: Why? What? How? *The Mathematics Teacher*, *94*(6), 442-446.

**Shaughnessy, M. (2012).** *STEM: An advocacy position, not a content area*. Presidents' Message, NCTM Summing Up.

**Silk, E. M., Higashi, R., Shoop, R., & Schunn, C. D. (2010).** Designing technology activities that teach mathematics. *The Technology Teacher*, *69*(4), 21-27.

**Silver, E. A. (1997).** Fostering creativity through instruction rich in mathematical problem-solving and problem posing. *ZDM*, *29*(3), 75-80. doi: 10.1007/s11858-997-0003-x

**Sternberg, R. J., & Williams, W. M. (1996).** *How to develop student creativity*. Alexandria, VA: Association for Supervision & Curriculum.

**Stuart, V. B. (2000).** Math curse or math anxiety? *Teaching Children Mathematics*, *6*(5), 330-335.

**Student Achievement Division. (2011).** *Maximizing student mathematical learning in the early years.* Capacity Building Series. Special Edition #22. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_Maximize_Math_Learning.pdf

**Suurtamm, C., Quigley, B., & Lazarus, J. (2015).** *Making space for students to think mathematically.* What Works? Research into Practice. Research Monograph #59. Student Achievement Division.

**Swank, A. L. G. (1999).** *The effect of weekly math homework on fourth grade student math performance*. (ED433234). Retrieved from http://eric.ed.gov/?id=ED433234

**Tamin, R.M., Bernard, R.M., Borokhovski, E., Abrami, P.C., & Schmid, R.F. (2011).** What forty years of research says about the impact of technology on learning: A second-order meta-analysis and validation study. *Review of Educational Research, 81*, 4-28.

**Tett, R. P., & Meyer, J. P. (1993).** Job satisfaction, organizational commitment, turnover intention, and turnover: path analyses based on meta-analytic findings. *Personnel psychology*, *46*(2), 259-293.TKCalifornia. (2016). Mathematics teaching strategies. A Project of Early Edge California. Retrieved from: http://www.tkcalifornia.org/teaching-tools/mathematics/teaching-strategies/?referrer=https://www.google.ca/

**United Kingdom Department for Education. (2012).** *Literacy and numeracy catch-up strategies.* Retrieved from: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/268031/literacy_and_numeracy_catch_up_strategies_in_secondary_schools.pdf

**Victoria State Government. (2009).** *Mental computation and estimation*. Department of Education and Early Childhood Development. Retrieved from https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/readmentalcompest.pdf

**Wagganer, E. L. (2015).** Creating math talk communities. *Teaching Children Mathematics*, *22*(4), 248-254.

**Wathall, J. T. (2016).** *Concept-based mathematics: Teaching for deep understanding in secondary classrooms*. Thousand Oakes, CA: Corwin Press.

**Whitcombe, A. (1988).** Mathematics: Creativity, imagination, beauty. *Mathematics in School*, *17*(2), 13-15.

**Wing, J. M. (2006).** Computational thinking. *Communications of the ACM*, *49*(3), 33-35.